OBJECTS AS LIMITS OF EXPERIENCE AND THE NOTION OF HORIZON IN MATHEMATICAL THEORIES

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1. Introduction

In this article I try to provide a solid argumentation on the relevance of key phenomenological notions, namely that of the life-world and that of (phenomenological) horizon with the philosophical motivation of certain alternative versions of standard set theory that have sprouted in the last 40 years mostly out of the impact that bore in foundational mathematics Gödel's incompleteness theorems, P. Cohen's proof of the independence of key infinity assertions (i.e., the Continuum Hypothesis, CH, and the Axiom of Choice, AC) and A. Robinson's classical *Nonstandard Analysis*. Among these alternative theories, the Alternative Set Theory (AST) of the Prague School of around the last quarter of 20th century explicitly claimed, by the words of one of its main representatives P. Vopěnka, its motivation as being based on Husserl's ideas in *Die Krisis der Europäischen Wissenschaften und die Transzendentale Phänomenologie*, ([2]). Criticizing the rigidity and dogmatic acceptance in Cantor's theory of the notion of actual infinity associated, for instance, with the declaration that all objects capable of successive, infinitely extending construction have already been constructed, P. Vopěnka claimed that:

> 'Thus we deal here with a construction extending the real world and surpassing qualitatively the limits of the space of possibilities of our observation. Assertions about infinite sets thus lose their phenomenal content [...] One possible way out of the crisis of contemporary mathematics may be through an attempt to reconstruct mathematics on a phenomenal basis [...] But a purely phenomenal conception of mathematics would considerably impoverish mathematics [...] Mathematics is a means for surpassing the horizon of human experience [...] In reconstructing mathematics we are thus obliged to accept also basic principles for surpassing the horizon of evidence.' ([15], p. 6 & p. 10).

A more detailed review of the phenomenologically 'relevant' content of AST theory will be undertaken in section 3. Also, in section 4 the same will be done with respect to another non-Cantorian and nonstandard theory thought of, under a certain semantical interpretation, as having a certain conceptual proximity with the notion of 'observable' horizon, namely, the Internal Set Theory (IST).

In my general approach a main purpose, next to pointing to the phenomenological affinities of the content of the theories above, is to present a strong argumentation for the assertion that infinity in general, in terms of a constituted whole surpassing the limits of evidence based on human experience, may be only embedded in the kind of mathematics conceived in the sense of a Husserlian mathematical natural science by means of infinity principles of *ad hoc* nature incorporating an implicit, at least, notion of actual infinity[1]. This means that in case we consider the horizon as the ever shifting limit of concatenations of human experience, to 'reach' or even 'transcend' the bounds of horizon we are set to fall into the same circular maze one is caught in Cantorian mathematics with regard to infinite objects in actuality. That is, we have to accept a disguised form of actual infinity principle to make the 'leap' from the ever advancing path of hereditary countability, in other terms from the progressing succession of concatenations of experience in implementing any 'observation' within the life-world, to the 'vagueness' of continuous unity upon or even beyond the horizon.

In section 2, I deal to some extent with certain core ideas of Husserlian *Krisis* especially in connection with my professed scope above and set about to propound the main pillars of Husserl's scheme to mathematize nature on condition of experiencing the world as being already there but, all the same, as being there for us awake to the world, directly (without any mediation) conscious of the world and of oneself, 'as living in the world, actually experiencing and actually effecting the ontic certainty of the world' ([2], p. 143). The world, in the sense of a world-horizon cannot be made a thematic object of phenomenological attitude as an object or entity but presupposes every act of positing objects within-it and consequently implies fundamentally different correlative types of consciousness with respect to them. This is a notion of the world as essentially an eidetic horizon conditioning the exact determination of mathematical objects as limit-idealizations in the construction of a mathematics

[1] The term actual infinity here and throughout the text is taken as somehow stronger than its usual connotation, i.e., the conception of infinity (mathematical) objects as completed wholes with a typical instance the conception of the set of natural numbers as a whole in actual presence. My view of actual infinity is more focused on the conception of uncountable infinity as the immanent unity of a whole in presentational immediacy.

based on the normativity of natural 'appearances' with regard to all carriers of a constituting consciousness in intersubjective coincidence. As such it stands as a key concept, naturally in relation to at least one consciousness intentionally oriented within it, of Husserlian *Krisis*.

It is true, though, that Husserl had made several important remarks on the broad notion of life-world as the possible world of experience that depends on the possibility of experience, in 'being' itself already and primordially there (thus avoiding the pitfalls of solipsism), already at the time of publication of *Ideas I*. It is notable that he had also used the term horizon to describe the 'correlate of the components of undeterminateness essentially attached to experiences of physical things themselves; and those components – again, essentially – leave open possibilities of fulfillment which are by no means completely undetermined but are, on the contrary, motivated possibilities predelineated with respect to their essential type'. In the same passage he went on to clarify that any actual experience points to a potentially infinitely extending concatenation of experiences which are effected 'involving species and regulative forms restricted to certain *a priori* types'; moreover, 'Any hypothetical formulation in practical life or in empirical science relates to this changing but always co-posited horizon whereby the positing of the world receives its essential sense' (engl. transl. [5], p. 107; [4], pp. 101--102). He even presaged the key to understanding his idea of life-world notion of intersubjectivity by referring to the 'existence' of an actual ego 'as a demonstrable unity relative to its concatenations of experience', to ground the validity and the essential possibility of determination of something transcendent to the world of real experience; in this position, what is cognizable by one ego must be by essential necessity be cognizable by any ego in the sense that there exist essential possibilities that the separated worlds of experience corresponding to each particular ego are in fact joined by concatenations of actual experience to identically make up the one intersubjective world as a correlate of the unitary world of human mental lives (engl. transl. [5], p. 108; [4], pp. 102-103).

In subsection 2.1, to strengthen my arguments on the essential necessity to recourse to some *ad hoc* actual infinity principle to shift the bounds of an ever advancing hereditarily finite horizon to the impredicative nature of continuous unity, I refer to some fundamental features of the phenomenology of consciousness. This is done in holding to the position that the possibility of the world is conditioned in intersubjective fashion on the possibility of each and every experience (in the essential mode it is carried through) constituting the world. In addition, one has to keep in mind that the universal

a priori of the logical-mathematical sciences, generally the objective-logical a priori, is founded on the universal a priori 'which is in itself prior', that of the pure life-world ([2], p. 141). As a matter of fact my reference to the phenomenology of temporal consciousness in subsection 2.1, to the extent that infinite mathematical objects or notions, e.g. a transfinite recursion or an infinite well-ordering, may be thought of as purely mental constructions, may be justified in an almost reverse sense to that of G. Longo in [10]. He claims there that 'infinitary constructions in mental space and time may be understood as the subjective traces of intersubjective extensions of the objectivity of the phenomenal world, i.e. they are the 'mental marks' of the objectivity we constructed in intersubjective, historical praxis, over basic regularities. The concept of actual infinity is the result of many historical conceptual constructions [...] Its objectivity is obtained as an integration of 'metaphores' ([..] but they are not linguistic metaphores) and by the normative structuring of mathematics, well beyond phenomena and leaves traces in our minds;' ([10], pp. 24-25). My own approach is that there is a purely subjective origin associated with the constituting and, ultimately, self-constituting modes of consciousness that makes possible the establishment of infinite totalities as completed wholes in the present now independently of spatio-temporal and causal constraints and yet 'motivated' by the modes of experiencing within the life-world.

Finally in subsection 2.2, I refer to a simplified mathematical model (by J. Petitot) in which kinesthetic descriptions are also ultimately conditioned, in the processional continuous limit, on a notion of actual infinity in the general sense of a completed whole in presentational immediacy.

2. Outline of the relevance of certain key notions of Husserl's *Krisis* with alternative mathematical theories

In *Krisis*, which eventually came to be Husserl's last published work (lasting from 1934 to 1937), the founder of phenomenology inquired about the possibility of attaining what is in itself true of nature by means of what he termed a mathematical natural science. On this account, he drew a distinction between an ontology of nature 'in itself', meaning the necessary forms for a determination of the ideal essence of nature as such and of every individual which *idealiter* and 'in itself' can belong to nature, (something that can be accomplished by the pure mathematics of nature), and an a priori methodology of a possible knowledge of nature in itself in taking nature as experienced by experiencing beings. This latter possibility, that is, the possibility of the

knowledge of nature in itself through nature experience begets the a priori possibility of a mathematical natural science, or 'the science of the method of natural-scientific determination of nature through the data of experience' ([2], pp. 305-306).

In this respect, Husserl devoted a significant part of this work to develop the notion of mathematical natural science as a new mathematics bearing its origin in Galileo's mathematization of nature in the sense that nature may be idealized by means of a new mathematics marking the rift with classical Greek mathematics that dealt only with finite tasks and thus referred to a finitely closed a priori body of (mathematical) knowledge. In contrast, the dawn of modern era saw mathematics becoming a rational universal science which can master the infinite totality of what in general may be taken as a rational all-encompassing unity, including also natural science insofar as it can be idealized by the new mathematical norms ([2], pp. 22-23). Husserl saw as the factually 'imposed' link to infinity the concept of geometrical space and geometry as the science belonging to it. To the ideal space, in his claim, belongs 'a universal, systematically coherent a priori, an infinite, and yet – in spite of its infinity – self-enclosed, coherent systematic theory which, proceeding from axiomatic concepts and propositions, permits the deductively univocal construction of any conceivable shape which can be drawn in space' ([2], p. 22). Generalizing the notion of infinite totality to that one attained by formal mathematics as a rational science proceeding apodictically through axioms, propositions, inferences, proofs, etc., Husserl described an infinite world of idealities whose objects are reached by a rational, systematically coherent method where in its infinite progression every object is ultimately attained in its full being-in-itself. However, he was eventually reserved concerning this last assertion, as in a subsequent part of *Krisis* he drew attention to the fact that the perceived being, the experienced as such, always stands 'under the essential law of a certain gradation of perfection which always exists as an ideal possibility' ([2], p. 309). Moreover he pointed out that correlatively to the differentiation of perfection are to be associated free 'can'-possibilities of approximation to the absolutely perfect, that is, the true in-itself of the object, even though he regarded it as forever receding. In this approach, the absolute in-itself of an object, in Husserl's view the identical self of the object through its multiplicity of appearances, together with its true characteristics are to be considered as limits of a possible gradation. It follows that insofar as mathematical characteristics are the only 'true' ones, true mathematical characteristics are utterly mathematical limits.

Overall, Husserl outlined an exact determination of mathematical objects by means of a limit-idealization which is conditioned on subjectively changing 'modes of appearance' of the objects in question and therefore entails the possibility of their re-identification as identical in-themselves and of their 'truthfulness', by virtue of being contents of a determinative thinking (e.g., of general judgements, inferences, proofs, etc.), as subjectively conditioned. In Husserl's words 'A certain idealizing accomplishment is what brings about the higher-level meaning-formation and ontic validity of the mathematical and every other objective a priori on the basis of the life-world a priori' ([2], p. 140). In this sense, all the laws of mathematics to the extent that they apply to the real world are particularizations of the laws of formal ontology insofar as this latter was described in earlier Husserlian works (primarily in *Formale und transzendentale Logik*) as a pillar of formal mathematics (in Husserl's theory of manifolds) and as generally referring to objects as registered by (intentional) experience. In such approach, a notion of absolute equality may be seen *idealiter* as the limit of converging processes something that in an approximation process presupposes a conception of an unchanging magnitude that is taken as an absolute measurement unit and which is moreover always identical with itself. Consequently in consistency with Husserl's original characterization of phenomenology as a kinematical science in contrast with classical ontology's 'static' attitude, fixedness and categorial properties of formal objects were defined as limit idealizations of kinetical processes that founded formal objects' ontology on an ever continuing subjectively conditioned synthesis. In view of this position, ideal objects (e.g., ideal spatial shapes) admit of an exact determination which means that in that case the law of excluded middle is valid, whereas empirically experienced objects do not admit of such determination since they can only be determined as being experienced in the present now independently of being objects of sensible or mental experience. As Husserl rightly pointed out, the experienced object I reflect upon now is not the 'same' with itself being experienced a moment earlier, consequently for empirically experienced objects does not hold the law of excluded middle ([2], p. 314).

Given the Husserlian claim for the need of a new sort of thinking, or a peculiar method, to extract objectivity and truth in itself as subjectively conditioned through experiencing within-the-world, I will attempt in the following to bring out the axiomatical structure and the semantical content of certain mathematical theories allegedly finding their philosophical motivation in several key concepts of Husserl's *Krisis* (AST theory) or having, under a proper interpretation, a certain relevance with these very concepts (e.g.,

IST theory and certain nonstandard theories, including intuitionist theory, dealt with in the next sections). More specifically, a special attention will be given, (1): to the notion of phenomenological horizon inasmuch as this may be conceived as the ever expanding boundary of experience within the life--world (the latter meant as the original and unconditioned 'ground' of every possible experience) and consequently as the foundation of the possibility of attaining a limit idealization of mathematical objects and ultimately of a rational infinite totality of being, and (2): to the notion of the shift of the horizon of natural intuition with its associated categorial objectivities by some extension principle which nevertheless is conservative with regard to the pre--horizontal 'environment' and (3): to the constraints put upon the possibility of an exact determination of the objects of a mathematical natural science by the subjective and 'local' character of the observations of corresponding subjects in intersubjective coincidence.

Concerning the capital notion of horizon, Husserl described it in *Krisis* in a way that is generally consistent with his longtime endeavor in transcendental phenomenology, namely the constituting role of consciousness after phenomenological *Epochë*. In this view, every perception (whatever might be its meaning) of an object is associated with a horizon of the object in question with regard to a consciousness .This is further clarified as, for example, a given object of perception[2] is as such never what it was a moment earlier, in the sense that is is given in a multiplicity of profiles corresponding to the multiplicity of its appearances in each present now of consciousness and yet it is constituted as identically one and the same through the continuous flux of consciousness. The perspectives of the 'exhibiting of' of each object 'combine in an advancing enrichment of meaning and a continuing development of meaning, such that what no longer appears is still valid as retained and such that the prior meaning which anticipates a continuous flow, the expectation of what "is to come", is straightway fulfilled and more closely determined. Thus everything is taken up into the unity of validity or into the one, the thing.' ([2], p. 158). Moreover, in the particular perception of a thing corresponds a whole horizon of nonactive and yet co-functioning modes of appearance and syntheses of validity which inescapably make that in the unfolding of the correlation we bear as constituting subjects to things, e.g. on the physical

[2] It should be noted here that Husserl regarded mathematical objects, in the sense of objects of mathematical theories, as close yet distinct to perceptual objects on the constitutional level. He termed the intuition of mathematical-syntactical objects as categorial intuition in their most fundamental sense of 'empty-somethings' in general, devoid of any content whatsoever, and as transformations of these 'empty somethings'; see, *Ideen* I and *Formale und transzendentale Logik*, resp. [4], pp. 33-34 & [8], pp. 91-92.

level on a closer inspection of an object at a certain distance, we realize that 'unnoticed limitations, horizons which have not been felt, push us on to inquire into new correlations inseparably bound up with those already displayed' (ibid. 159). This is a point of a special importance regarding the aforementioned mathematical theories inasmuch as it establishes a notion of 'internal horizon', referring in particular to non-finitistic mathematical objects (or processes), that reflects the possibility of unfolding properties in a systematic multiplicity of all possible 'exhibitings of' or yet inherent limitations in the unfolding of all possible 'appearances' of the objects in question. However, the 'internal horizon' cannot but be tied up to the 'external horizon' establishing any individual thing within a field of things and ultimately to the whole world as perceptual world and universal horizon of experience. Moreover, the notion of an 'internal horizon' of things-in-the-world is based on a correlation of the unfolding profiles of a thing within horizon with corresponding kinestheses of an 'acting' subject in a way that there may be a 'stable' consciousness of one and the same thing in actual presence exhibiting itself in a multiplicity of modes of 'appearance'. The simplest case of an intentional analysis of perception through the kinesthetic capacities of a subject is that of a thing remaining at rest and being qualitatively unchanged.

As I will show in the next, there will be need of some form of actual infinity notion to mathematically represent an exact determination of a mathematical object meant as a definite whole. Further, as it will also be seen in the exposition of the underlying semantical content of certain key notions of the mathematical theories to be dealt with, some form of actual infinity principle (e.g., the Prolongation Axiom in AST theory) is applied to 'bridge' the countable path to infinity with uncountable infinity, the latter concept serving as a set-theoretical foundation for the notion of mathematical continuity in being conceived as a completed whole in the present now of reflection. In terms more close to their phenomenological content in *Krisis*, the application of an actual infinity principle may serve to tie up the path of multiplicity of 'appearances' of an mathematical object (or concept)[3] toward the bounds of its 'horizon', with its limit-idealization upon the 'vagueness of horizon'. In essence, this notion of horizon marks the limits of our intuitional capacities in intersubjective coincidence within-the-world, at least of our possibility as performers of the intentional correlation *original impression--retention-protention* to further constitute in noetical-noematical fashion well-

[3] A mathematical object (or concept), e.g. a topological surface, is taken here as the formal counterpart of its 'real' existence in the act of 'observation' of a locally interacting and intentionally oriented embodied consciousness within the bounds of the life-world.

-defined objects in phenomenological perception (*Wahrnehmung*). As a matter of fact, in a kind of circular play, an actual infinity principle in the form of an *ad hoc* extension (prolongation) axiom is warranted to shift the horizon of countably infinite processes to the 'vagueness' of continuous unity conceived of as the ultimate qualitative shift of a horizon beyond which possibly collapse all our intuitions ([16], p. 123).

In the immediately following sections I'll draw attention to certain concepts in Husserl's description of the phenomenology of consciousness and, next, to certain features of a simple mathematical modelization of the notion of phenomenological kinesthese to point to the inherent necessity to turn to some 'higher-order' actual infinity principle to 'accede' to the horizon of limit-idealization of (mathematical) objects themselves in the sense of their constitution as complete totalities in the present now of reflection. In this concern it is essential to take account that the possibility of the world of experience for Husserl is conditioned on the possibility of experience itself which gives the world its reality and sense; in short, it is the experience of a subject that constitutes the world in intersubjective coincidence ([4], pp. 102--103). Moreover, it must be noted that Husserl never retreated from his longstanding position on the reduction of the being of the world and of every possible object within-the-world to the ultimate subjective origin of their constitution as such which is the transcendental ego of consciousness.

2.1 Temporal aspects of the radical phenomenological reduction

According to Husserl's claim in *Ideas III*, the phenomenological analysis is of a kinetical (*kinetisch*) and not of a catastematical (*katastematisch*) character[4]. Moreover the conviction to an objective reality in an absolute sense is put in suspense – the corresponding well-known term is Epochë – by a constituted reality approach which is fundamental in phenomenological analysis. The constituted objects become immanent[5] to the constituting flux

[4] The meaning of the term catastematical should be taken here to be the same as ontologically hypostasized. The following Husserlian quotation helps to better comprehend the dierence between the aforementioned terms: "The mode of ontological consideration is, so to say, catastematical. It takes the unities in their identity and with regard to their identity as something fixed. The phenomenological and constitutive consideration takes such a unity in the flux, which means in terms of a unity of the constituting flux; it is attached on the movements, on the flows in which such a unity and every component, aspect or real property of this unity is correlate of the identity." (transl. of the author, [7], Beil. I, p. 129).

[5] This is a phenomenological term attributed to an object which is no more transcendent to an intentional consciousness, i.e. an object of 'external' reality, but has been modied to a noematical correlate of the constituting flux of consciousness.

of conscience in which they are immanentized in a certain mode, that is, in terms of certain retentional forms of the constituting flux; for instance, in the *vor-zugleich* (anterior-simultaneous) mode of retention which generates a continuous sequence of retained phases trailing behind an original impression each of which is a retentional consciousness of the preceding one, and this way for each new original impression ([6], pp. 29-33).

The temporal consciousness of immanent objects is the unity of a whole, an all encompassing unity of retentions of original impressions apprehended in actuality which modifies continuously the multiplicities of original impressions into a trailing sequence of just-passed-by retentions together with the retentions of these retentions just-passed-by and so on, in a way that one can talk about a double or longitudinal intentionality (*Längstintentionalita¨t*) of the constituting flux of conscience. By this specific intentional form, Husserl meant the retention of an immanent object as such in consciousness, e.g. a sonorous effect in the present now, and also the consciousness of the retention of this sonorous effect as such, constituting by this token its immanent unity with the sequence of all former phases preceding this effect in terms of a continuous whole within the homogenous flow of the flux ([6], pp. 80-81). In Husserl's words: "The totality of the group of original impressions is bound to this law: It transforms itself into a constant continuum (*in ein stetiges Kontinuum*) of modes of consciousness, of modes of being-in the flow and in the same constance, an incessantly new group of original impressions taking originally its point of depart to pass constantly (*stetig*) in its turn in the being--in the flow. What is a group in the sense of a group of original impressions remains in the modality of being-in-the flow." (*transl. of the author*, [6], p. 77).

Consequently, by appealing to the double intentionality of the absolute flux of consciousness Husserl posited the immediate retention of an immanent object in the flux of conscience (the sonorous effect of a sound, for example), on the one hand, and the constitution of a 'descending' sequence of retentions of the original impression of this object, on the other, as a continuous unity always in the anterior-simultaneous mode of the flow; "Thus, the flux is traversed by a longitudinal intentionality which, in the course of flux, overlapps continuously with itself." ([6], *transl. of the author*, p. 81).

However, in the retentional-protentional and longitudinal[6] mode of the

[6] The retention (or primary memory) and the protention are phenomenological terms which are meant as specific intentional modes of consciousness; respectively, as an a priori in character immediate conservation in memory of the immanence of an original impression and the a-thematic attendance toward a not-yet-perceived impression. These are described by Husserl in terms of the transversal intentionality (Querintentionalität) of consciousness,

self-constitution of the flux there is no irreducible definition of the term continuity used by all accounts in a somehow circular sense in terms of the constituted (by the intentional modes) unity of the flux. Moreover, the self--constitution of the flux as a phenomenon in itself is not but an objectification of what is thought to be the ultimate subjectivity, the absolute ego, in other words the absolute subjectivity of the flux of consciousness. This is a supplementary and most radical phenomenological reduction, probably the key to comprehend the inherent vagueness of the notion of continuity even in the kinetical terms of the constituted reality in Husserlian sense. For, Husserl himself claimed that it is impossible to extend the phases of the absolute subjectivity of the flux in a continuous succession, to transform it mentally in a way that each phase 'extends' identically on itself, a certain phase of it belonging to a present that constitutes or to a past that also constitutes (not constituted), to the degree that it is an absolute subjectivity beyond any predication and whose retentional continuity in the constituting flux is not but its objectification, its ontification by its 'mirror' reflexion ([6], pp. 73-75).

It is clear that what is being intuited as a continuous flow in the temporal constitution of a group of simultaneities and corresponding retentions is irreducible in terms of an ontological deconstruction to constituent non--durating parts as it is essentially the objectification of an inherently elusive process which is always 'on the go', and where every attempt to simply reflect on it produces its 'mirror' objectification. The underlying role of this ultimate irreducibility within the flux of temporal consciousness which, in my view, is reflected in the inherent impredicativity of intuitive continuum and even further (under a certain interpretation) of mathematical continuity, I'll try to put in evidence in the axiomatical foundation of Alternative and Internal Set Theories as non-Cantorian versions of nonstandard theories in sections 3 and 4.

More to the point, I argue that there must ultimately be an immanent subjective root within each one's consciousness to account for the possibility to reach continuous unity of non-finite mathematical-logical objects as definite wholes solely through limit-idealizations (e.g., by applying some sort of actual infinity principle). Let us keep also in mind what H. Weyl stated in *Das Kontinuum*, namely that 'as basic a notion as that of the point in the continuum lacks the required support in intuition. It is to the credit of Bergson's philosophy to have pointed out forcefully this deep division between the world of mathematical concepts and the immediately experienced continuity of phenomenal time ("la durée").' ([17], p. 90).

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in the a priori scheme original impression-protention-retention. For details the reader may consult the Husserlian texts in [6], (resp. pp. 29-33 and pp. 52-53).

From the unfolding of my arguments in the next sections, it will also become clear how the theories above offer a more natural mathematical approach to real processes in life, as at least P. Vopěnka claimed that AST theory does. In general, AST tries to imitate real processes in a witnessed and mutually interacting universe by following the unfolding of hereditarily finite multiplicities of phenomena to the horizon of 'observability' and by postulating, through *ad hoc* axiomatization, vagueness beyond the horizon of 'observability'. It is more or less along this general conceptual approach that non-Cantorian theories as well as intuitionistic ones follow an alternative path to the notion of continuum.

2.2 Vagueness in phenomenological kinesthesia

The problem of the kinesthetic control of perception belongs to "the great question [..] of penetrating as deeply as possible into three-dimensional phenomenological 'creation', or, in other words, into the phenomenological constitution of the identity of the 'body' of a thing through the multiplicity of its appearances" ([3], *transl. of the author*, p. 154). It also points, as will be seen in a specific case below, to a possible interpretation of the passage from empirically experienced spatial shapes to ideal ones as referred in Husserl's *Krisis* (p. 314).

The kinesthetic control of perception is not only a presupposition for the effective identity of an appearing object, thus founding logical identity upon continuous variability and synthetic *a priori* laws to a continuous synthesis (which is, in fact, a kinetical synthesis); it also "rules phenomenologically temporal series corresponding to three classes of movements, namely, those of the eyes, of the body and of objects" ([12], p. 354). Kinesthetic control is essential too, in interpreting phenomenologically the source of each movement as something 'internal' to the Husserlian kinesthetic sensations. As I will proceed now with the kinesthetic analysis of the simplest situation, which is that of a body (subject) being fixed and the object/s remaining at rest, it will be shown at least on the formal level, that 'vagueness' in the continuous limit underlies the unity of the constituted movements even though it is based on the temporal discreteness of the correlations $k_1 \leftrightarrow i_1,.., k_n \leftrightarrow i_n,...$ The particular situation consists in the purely ocular kinesthetic sensation schematized by the correspondence $k \leftrightarrow i$ between the space of kinesthetic controls K and the space of visual images F in applying a temporal parametrization through reciprocally corresponding paths k_t, i_t.

In [12], J. Petitot refers to an elementary model from the theory of (geometrical) manifolds, to discuss the nature of the association between

k_t and i_t (the temporal paths of kinesthetic sensations and those of image variations) and also that of the 'fixed association' of the space of kinesthetic controls K with the visual field M, the latter modelized as a simple domain D (a two-dimensional disk). We may imagine the domain D as a geometrical square S with end-points a, b, c, d, where to each end-point $p = a$, $p = b$, $p = c$, $p = d$ corresponds a 'slice' Dp of the field D as a way of interpreting the focusing on each such point.

Quoting from Jean Petitot: "If the figure i_a filling in D_a can 'refer' to the figure i_b filling in D_b, it is because D_a and D_b overlap and are glued together through their intersection $U_{ab}=D_a \cap D_b$. This means that there exists a local gluing isomorphism $\varphi_{ab}:U_{ab}\subset D_a \rightarrow U_{ab} \subset D_b$ identifying the intersection U_{ab} viewed as a subdomain of D_a with the same U_{ab} viewed as a subdomain of D_b. In the continuous limit, there exists a temporal series D_t with gluing operators $\varphi_{tt'}$ for t and t' sufficiently near. This spatiotemporal series is filled in by the image series i_t. To say that the 'pointing' of each i_t to another $i_{t'}$ is intentional or that intentions 'go through' the series i_b, is to say that intentionality corresponds to gluing operators identifying different points of the visual flow as the same [...] More precisely, intentionality corresponds to the realization in consciousness of the gluing operators. Once again, it is essential here not to confuse, as the natural attitude does, the constituting level and the constituted one [...] This is the main role of kinesthetic controls: the k_t are gluing protocols." ([12], pp. 356-357).

The formalization of the kinesthetic constitution of movement, purely ocular in our instance, by means of gluing operators k_t realized in consciousness for t, t' sufficiently near, may be seen as a representation in a mathematically meaningful language of the concept of intentionality of the constituting flux of consciousness. As it was the case with the retentional modes of consciousness, namely the transversal and longitudinal intentionality, it is also clear that in the phenomenology of movement through kinesthetic controls one cannot avoid the circular introduction of the notion of continuity in the description of a constituted unity out of the multiplicity of kinesthetic controls. In the present case, the continuity factor conditioned on the implicit acceptance of an actual infinity, is represented by the local gluing isomorphisms $\varphi_{t,t'}$ for t and t' sufficiently near that 'glue' together the temporal series D_t filled in by the image series i_t as a continuous whole of constituted reality. Concerning the gluing isomorphisms $\varphi_{t,t'}$ in the mathematical model above, I draw a parallel in the pure phenomenology of consciousness with the 'conjunction' of a sequence of immanences of original impressions in the flux of consciousness

(with the descending tail of retentions in-between) constituted as a continuous unity by the intentional modes of the constituting flux and ultimately by its non-objectifiable subjective origin (pure ego).

At this point, prior to dealing with indiscernibility or vagueness from the standpoint of Alternative and Internal Set theories in sections 3 and 4, it is important to refer to the Husserlian idea of scale invariance, as evident generic similarity which can lead to minima visibilia as point-like ultimate minimalities bearing the same eidetic relationships 'discovered' in the macroscopic universe, ([3], p. 166).This idea seems to have a profound effect on a shift of the horizon principle embodied in AST theory.[7]

3. The phenomenological relevance of the AST approach

The Alternative Set Theory (AST), as it happens with other nonstandard and (so-called) non-Cantorian versions of standard (ZF) set theory, was born out of the theoretical doubts raised in foundational mathematics during the 20th century and especially in the fermentation that followed Gödel's incompleteness results and the attempts to develop alternative formal theories that were willing to dispense with 'sacred' Cantorian principles. What makes, however, AST relevant to the scope of this article is that its conceptual motivation, as it is represented in the axiomatical construction, is explicitly based on the core ideas of Husserl's *Krisis*. As the main representative of the Prague School of this theory P. Vopěnka stated in describing the notion of countability in terms of AST: 'Our capacity for observation and distinction is limited by the horizon in all directions. Needless to say, this applies not only to optical observation; the horizon is understood in the sense of E. Husserl's *Krisis der Europäischen Wissenschaften und die Transzendentale Phänomenologie*.' ([15], p. 39). Yet, as it happens with standard set theory in the context of its own formal-predicative universe, at some point AST has to face the question of the axiomatical 'incorporation' of the transcending of the horizon beyond the limits of well-meant perceptional capabilities within the life-world. As it will be shown below this is done in taking recourse to a general extension principle (i.e., the Prolongation Axiom) that is basically conditioned on the acceptance of a certain notion of actual infinity and is conservative in character in that it leaves intact the essential character of

[7] This generic similarity justies the transposition of the eidetic relationships 'discovered' in the universe of common intuition to that beyond the horizon. It is remarkable, though, that P. Vopěnka seems to deny this principle in [16], (p. 123), where he insists that all ideas held hitherto could collapse beyond some genuinely qualitative shift of the horizon.

the entities 'existing' beyond the horizon with respect to their restriction in advance of the horizon.

One of the key features of AST's alternative theoretical approach is the reduction of the continuity of topological shapes and motions to the extension, by the application of the axiom of prolongation, of finite segments of the class of natural numbers to class infinities transcending the 'horizon of observation'. In this sense, a topology can be defined relying basically on the notion of countable classes in the extended universe of sets and on the principle of prolongation. Therefore, one need not adopt the traditional approach of topological openness, connectedness, etc. fundamentally based on the inner structure and generated continuity of the real numbers system.

The Alternative Set Theory, as exposed in its fundamentals by P. Vopěnka in [15], determines a universe of sets formed by sets constructed iteratively from the empty set together with some axioms subjecting the sets of this universe to laws valid in Cantorian set theory for finite sets, excluding 'abnormal' circularities like the set of all sets. The universe is extended by the inclusion of classes of the form $\{x; \varphi(x)\}$ where $\varphi(x)$ is a property of sets from the universe of sets; classes that are not sets are called proper classes such as the universal $\prod$-class of the theory. On the formal-syntactical level, AST basically deals with sets and classes as objects. Sets are definite (may be very large), sharply defined and finite from the classical point of view, taking into account that in its universe of sets AST accepts the axioms of Zermelo-Fraenkel theory with the exception of the axiom of infinity. Classes, on the other hand, represent indefinite clusters of objects (possibly found inside very large sets) such as the class N of natural numbers in the classical sense. Therefore, the extended universe of the theory includes some extra axioms, in addition to those of the well-known universe of sets V, which are not set-theoretical formulas and are presented below (p. 13). It is remarkable that countability in the sense of hereditary finiteness is closely related to the notion of 'observation' toward the 'horizon'.[8]

In giving a formal definition of AST-countability below, we need know in advance that: Segment of a class A is called a subclass of the original class with respect to a linear ordering which contains with each of its elements all its predecessors.

Formally one has two definitions:

– *A pair* $(A, \leq)$ *of classes is called an ordering of type* ω *iff:*

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[8] By the words of P. Vopěnka, 'If a large set x is observed then the class of all elements of x that lie before the horizon need not be innite but may converge toward the horizon. The phenomenon of innity associated with the observation of such a class is called countability' ([15], p. 39).

1. *≤ linearly orders A*
2. *A is infinite and*
3. *for each $x \in A$, the segment $\{y \in A; y \leq x\}$ is finite.*

– A class X is called countable iff there is a relation R such that (X, R) is an ordering of type ω. A class is uncountable iff it is neither countable nor finite.

In case we go beyond the horizon of countability, in other terms beyond the 'horizon of observability' in a sense close to the notion of horizon in *Krisis*, AST theory adopts the following Prolongation (or shift of the Horizon) Axiom:[9]

For each countable function F there is a set function f such that $F \subseteq f$.

In a phenomenologically motivated interpretation, a remarkable consequence of the Prolongation Axiom is this: If a perceived (think of it as definable inside a intersubjective universe including at least one 'observer') state of affairs φ holds of every element of a sequence (x_n), $n \in \omega$, (where ω is the cardinality of countable infinity in AST) progressing toward the horizon, then (x_n), $n \in \omega$, is extensible to a sequence (x_β), $\beta \leq \alpha$ which crosses the horizon and its members also satisfy φ ([13], p. 394).

Put in a somewhat less formal language, any collection of elements perceived in a phenomenological sense as an aggregate of individuals-substrates of intentional observation 'in front of the horizon' (of AST countability) can extend beyond the horizon preserving the essential characteristics of its elements (e.g. individuality, ordering).

It is important to point out that P. Vopěnka made a fundamental distinction between the class of all finite natural numbers *FN* proved to be a countable class and the settheoretically definable proper class *N* of all natural numbers which is uncountable. In a witnessed universe, i.e., one that adopts the viewpoint of an observer of an extensible 'horizon' incorporated in it "The classical natural numbers correspond to the elements of *N* , whereas *FN* forms a canonical representative of the way to the horizon." ([15], p. 63).

As aforesaid, the extended universe of sets of AST theory includes two extra axioms which are not set-theoretical formulas:

[9] As Vopěnka's claimed: 'People have always tried to go beyond the horizon; this is a typical human aspiration. The aim is not merely to shift the horizon further away but to transcend it in the mind. Mathematics is one of the most important instruments for this; it formulates exact statements which transcend the framework of perception. We shall incorporate a typical principle of transcending the horizon into our theory in the form of an axiom (*author's note: the Prolongation Axiom*).' ([15], p. 41).

The axiom of existence of classes:

For any property $\varphi(x)$ of sets in the universe of sets the extended universe contains the class $\{x; \varphi(x)\}$

The axiom of existence of proper semi-sets: There is a proper semi-set. In AST language, $(\exists X)\ (\mathrm{Sms}(X) \wedge \neg\mathrm{Set}(X))$

Proper semi-sets play a very important role in the axiomatical construction of AST as they are thought of to represent classes inside sets and, roughly, blurriness and nonsurveyability in the 'observation' inside very large sets.

In this formal-axiomatical context one can define a topology by the Kuratowski closure operations which are not taken as primitive as it is the case in general topology, but they are instead defined in terms of an indiscernibility equivalence, $\doteq$, which underlies every topological definition and is fundamentally based on the principle of the shift of 'countability horizon', that is, on the prolongation axiom in the AST sense. The underlying idea in the definition of an indiscernibility equivalence is that in each infinite set of 'observed' objects there must be at least one pair (x, y) of mutually indiscernible elements; in mathematical form $x \doteq y$. As a matter of fact, a topology may be defined in the extended AST Universe with the notion of indiscernibility as a conceptual and formal foundation for all subsequent topological constructions, including the definition of a formal notion of motion ([15], pp. 87-88 and pp. 98-108). In short, indiscernibility relations in AST formalize vagueness or 'blurring of the vision' in topological structures occurring as we transcend, e.g., the horizon of countability of finite natural numbers to the uncountability (beyond the 'horizon') of the infinite proper class of natural numbers.

As it is already claimed AST, even in its *Krisis*-themed conceptual approach and its original insight in formally treating infinite mathematical objects (or state of affairs), cannot dispense with the inherent need to abridge the 'vague residuum' in-between the discrete, progressing succession of perceptual acts within the bounds of horizon and the unity conditioning limit--idealizations beyond the horizon (the latter in its universal *Krisis* sense). This constraint is met by incorporating the axiom of prolongation (together with those of the existence of proper classes and of semi-sets) as essentially *ad hoc* meta-axiomatical statements in the AST universe of sets. Nevertheless, it points to a question of a deep meta-theoretical significance that also concerns standard set theory and whose content can well be a subjectively rooted one, possibly referring to the constitutional capacities of each individual consciousness in intersubjective coincidence. But this is a wider discussion concerning also transcendental phenomenology and the phenomenology of

temporal consciousness which, (even though some hints were given in section 2.1), goes deep enough to 'transcend' the limits of this article.[10]

4. The Internal Set Theory's approach to continuity-vagueness

In this section I set out to outline the main conceptual and axiomatical characteristics of Internal Set Theory (IST), concerning in particular the key ideas of continuity and vagueness, in view of its adoption of an external to ZFC Set theory[11] undefined predicate *standard* involving indirectly the presence of an observer within classical Cantorian universe. As a matter of fact, the intensional development of a large part of nonstandard analysis mainly coincides with E. Nelson's Internal Set Theory (IST) appropriately interpreted, along with other nonstandard and non-Cantorian theories such as the Alternative Set Theory (AST) dealt with in the previous section, ultrafinitist theories (J. Hjelmslev, S. Lavine & A. S. Yessenin-Volpin), and more recently Nonstandard Class Theory and the Theory of Hyperfinite Sets. We must have clear that in the intensional development of nonstandard analysis, infinitesimals and infinitely large numbers do not exist in an objective way as in the extensional case (e.g. A. Robinson's nonstandard theory), but their existence has a rather subjective meaning and is related to the observational limitations of an interacting 'observer'. As a matter of fact, the introduction of the undefined predicate *standard* in Nelson's theory is metatheoretically associated with a factor of vagueness with regard to a series of 'observations' carried out in a discrete mode. It is suggested, for instance, that: 'finiteness' + 'vagueness' = 'unlimited', where 'unlimited' is a non-Cantorian equivalent to infinity.

In general, we define a vague predicate R with regard to a series of 'observations'
$O_0, O_1, \dots O_n$ as following:

1. $R(O_0)$
2. $R(O_i) \leftrightarrow R(O_{i+1}),\ i = 0, 1, \dots, n-1$, that is, O_i and O_{i+1} are indistinguishable with respect to R, and,
3. $\neg R(O_n)$

By applying the Transfer Principle of IST and appropriate theorems within the IST extended axiomatical system one can prove that the predicate

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[10] On this account, expect a forthcoming article of the author.

[11] The abbreviation stands for: Zermelo-Fraenkel Set Theory with the Axiom of Choice.

standard – abbreviated as *st* – is a vague predicate within the set N of natural numbers, ([1], pp. 295-296).[12]

From a syntactical point of view, E. Nelson introduced in the classical ZFC theory the new unary undefined predicate *standard* together with three axioms, the Transfer (T), the Idealization (I) and the Standardization (S) principles ([11], pp. 3-11). The *ad hoc* axiomatical machinery of the new predicate *standard* consists precisely of these three axioms, which in spite of their syntactical role in the theory induce *in rem* a nonstandard extension in the domain of 'fixed' objects. The term 'fixed' (in a broad sense finitistic) can be used as the intuition of the new predicate *standard* in informal mathematical discourse and it is implicitly associated with a domain of concrete 'observations' on the way to the horizon.

At this point it is important to refer to the intuition behind the idealization principle which alongside the transfer and standardization principles are the three extra axiomatical pillars of Internal Set Theory: 'The intuition behind the idealization principle is that we can only fix a finite number of objects at a time. To say that there is a y such that for all fixed x we have property A, is the same as saying that for any fixed finite set of x's there is a y such that A holds for all of them'; A is an internal predicate formula, that is, one that does not involve the 'unknown' predicate *standard* even indirectly

$$\forall^{st\,fin} x' \; \exists y \; \forall x \in x' \; A \longleftrightarrow \exists y \forall^{st} x \; A$$

([11], p. 5).

As it stands, we can deduce vagueness along infinity with respect to the set of natural numbers, in terms of the predicate *standard*, by relying on the idealization principle. Therefore we can point to a common conceptual underlying basis between the idealization and transfer principles of IST theory, on the one hand, which induce nonstandard elements[13] and the prolongation principle of AST theory, on the other, in the sense of an axiomatic means to 'shift' the horizon of phenomenological observability to the vagueness of continuum. It is to be noted here that although E. Nelson insisted that the predicate *standard* has a syntactical rather than a semantical role in the theory, it may be de facto taken as having a semantical content by the adoption of the

[12] In the proof of this theorem it is applied a straightforward result of the idealization principle of IST theory with vast semantical consequences, namely that every infinite set contains a nonstandard element. In particular, there exists a nonstandard natural number ([11], pp. 5-6).

[13] The Transfer Principle essentially states that if something is true for a fixed but arbitrary x, then it is true for all x: $\forall^{st} t_1 \ldots . \forall^{st} t_n \; [\forall^{st} x \; \mathbf{A} \leftrightarrow \forall x \; \mathbf{A}]$, where **A** is an internal formula (i.e., one that does not contain the 'unknown' predicate standard among its variables) whose only free variables are *x*, *t*1, *t*2, ..., *tn*.

three extra axioms. The convergence in the conceptual motivation is all the more evident to the extent that while in AST one may define topological notions with indiscernibility equivalences taken as primitive, in IST infinitesimality and unlimitedness (and hence continuity and openness) are defined by taking as primitive the predicate *standard* together with the I,T and S principles. This means that continuity and topological openness are not necessarily associated to the real number system as standardness and nonstandardness are not describable solely within the real model *R*. As a matter of fact, the novelty of the IST approach to topological continuity and openness stands in that it treats these fundamental ideas of analysis and topology by enriching the existing ZFC axiomatical system with the undefined and implicitly related to the local horizon of an 'observer' external predicate *standard* along with the appropriate axiomatical equipment with no reference, by necessity, to any particular mathematical model. In a certain sense, both the transfer and idealization principles may be viewed as axiomatizing the 'passage' either way from the the 'fixedness' associated with abstractions of life-world experiences carried out in progressing multiplicities of meaning-acts to the indefiniteness associated with constituted unity within the immanence of consciousness.[14]

In conclusion, there seems to be a common ground in the conceptual foundation of AST and IST theories that consists in the 'shift' of the bounds of hereditarily finite countability (AST) or of standard fixedness (IST) to the vagueness of infinity by the adoption of certain *ad hoc* axioms or predicates that may be thought of as 'external' to their first-order axiomatical system. This is essentially the case with the intuitionistic view, too. In both L.E.J. Brower's and H. Weyl's approach, the continuum is formally postulated by axiomatizing the shift to an indefinite horizon in terms of *ad hoc* extension principles beyond the natural bounds of the finite and discrete which, in the case of choice sequences, is represented by their initial segments. This axiomatization is primarily expressed by intuitionistic continuity principles such as L.E.J. Brower's *Continuity Principle for Universal Spreads* and H. Weyl's *Principle of Open data* ([14], pp. 220-224).

Lastly, we should take into account that in the standard Cantorian mathematics the deeper impredicative nature of vague 'beyond the horizon' infinity manifests itself, for instance, in the independence of certain key infinity statements such as the *Continuum Hypothesis* (CH) and the Axiom of Choice (AC) from the other axioms of ZF, something that is still a topic of hot theoretical debate among set theorists. In any case one has to appeal to actual

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14 For more details on the phenomenological relevance of nonstandard theories theory see: [9], (pp. 124-134).

infinity assumptions in one or other form, e.g. by performing second-order quantification on all subsets of the set of natural numbers N as completed wholes in the proof of the consistency of the negation of CH, to do meaningful mathematics in dealing with infinite structures. Only that in Cantorian mathematics and in a sense of platonistic realism, mathematical objects are taken as complete idealizations with no reference to their sources in real life processes and consequently as standing there in a realm inaccessible in its entirety to the human mind. In contrast, the alternative nonstandard theories I dealt with above are to one or other degree grounded on the admission of a notion of 'observation' on the part of a knowing and locally interacting subject within the bounds of the life-world in the sense this latter is provided as a pregiven carrier of a transposable horizon of intentions and meaning-acts.

5. Conclusion

In this article, I tried to bring out the conceptual foundation of certain mathematical theories based, on the one hand, on the phenomenological notion of the immanentization of multiplicities of appearances in the self-constituting unity of consciousness and, on the other, on the notion of an indefinitely extensible phenomenological horizon with respect to the intuition of discrete multiplicities of concatenating experiences. The extra axiomatization of these theories to describe the transcending to the vagueness of continuous unity stands essentially in the adoption of certain *ad hoc* 'external' axioms or predicates in addition to the standard axiomatical machinery of the classical Cantorian system.

Each of the theories mentioned in the sections above, with Alternative Set Theory in a more manifest way, attempts to formalize the vagueness inherent in the subjective constitution of an ideal 'reality' in the 'upper' limit of concrete real-world perceptions, by adopting a phenomenologically motivated attitude in the description of the horizon toward vague continuum and its underlying indiscernibility. As it turns out, they bear a measure of vagueness inherent in the intuition of infinity concepts in the field in which they become meaningful, that is, the field of our intersubjective life-world (*Lebenswelt*) in its ever shifting horizon. This is about a vagueness that reflects our inherent limitations to describe continuity in analytical terms and handle it mathematically in the same first-order language (without adding extra *ad hoc* axioms or undefined predicates) as that describing a hereditarily finite countability in our witnessed universe. More, this article also offered some clues to a deeper subjective origin of our capacity to create the unity of an ideal mathematical-logical

'world' out of the regularity of real world experiences. As G. Longo put it in [10], (p. 22): 'As for geometry, and following Riemann, Poincaré, Weyl, we referred to symmetries, isotropy, continuity and connectivity of space, regularities of action and movement, as "meaningful properties". They are meaningful as they are embedded in our main intentional experience as hinted above: life.'

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ABSTRACT

The present work is an attempt to bring attention to the application of several key ideas of Husserl's *Krisis* in the construction of certain mathematical theories that claim to be alternative nonstandard versions of the standard Zermelo-Fraenkel set theory. In general, these theories refute, at least semantically, the platonistic context of the Cantorian system and to one or the other degree are motivated by the notions of the lifeworld as the pregiven holistic field of experience and that of horizon as the boundary of human perceptions and the de facto constraint in reaching limit-idealizations. Moreover, I try to give convincing reasons for the existence of an ultimate constitutional 'vacuum' of a subjective origin that is formally reflected in the application of a notion of actual infinity in dealing generally with the mathematical infinite.

Keywords: Actual infinity; alternative theory; field of experience; horizon; life-world; limit-idealization; nonstandard.